Chair of Network Architectures and Services
School of Computation, Information and Technology
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Network Coding

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Tutorial 5

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Problem 1 Lossy wireless networks

We consider the three-node wireless relay network $G = (N, H)$ depicted in Figure 1 and the respective induced graph $G' = (N, A)$ in the lossy hypergraph model with orthogonal media access. For clarity, only the maximum hyperarcs are drawn in the figure.

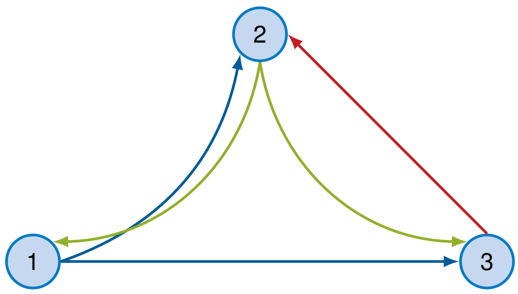

Figure 1: Three-node relay network

Hint: You may use the pre-printed Table 1 at the end of this problem sheet.

a)* Explicitly state the set of hyperarcs $\mathcal{H}$.

b) State the set of hyperarc indices H by numbering the hyperarcs $(a, B) \in \mathcal{H}$ in lexicographic ascending order, i. e., $(a, B) < (a', B')$ if

1. $a < a'$ *or*
2. $a = a' \ \wedge \ |B| < |B'|$ *or*
3. $a = a' \ \wedge \ |B| = |B'| \ \wedge \min B \setminus B' < \min B' \setminus B$,

such that $j \equiv (a, B)$ with $j \in H = \{1, 2, ...\}$ for all $(a, B) \in \mathcal{H}$.

c)*

For each $j \equiv (a, B) \in \mathcal{H}$, state the set $\mathcal{A}_j$ of arcs and corresponding indices A_j induced by j. Number the arcs $(a, b) \in \mathcal{A}$ in lexicographic ascending order, i. e., $(a, b) < (a', b')$ if

1. $a < a'$ *or*
2. $a = a' \ \wedge \ b < b'$,

such that $k \equiv (a, b)$ with $k \in \{1, 2, ...\}$ for all $(a, b) \in \mathcal{A}$.

d) Draw the graph $G' = (N, A)$ that is induced by G.

e) State the hyperarc-arc incidence matrix $\boldsymbol{N}$.

f) State the incidence matrix $\boldsymbol{M}$ for G'.

Assume that each arc $k \in A$ has unit capacity and a link error probability of $0 \leq \epsilon_k \leq 1$.

g) Determine the hyperarc capacity region

$$\mathcal{Z} = \bigcup_{\substack{\boldsymbol{\tau} \geq \mathbf{0} \\ \mathbf{1}^T \boldsymbol{\tau} \leq 1}} \left\{ \boldsymbol{z} : z_j = \tau_a \prod_{k \in A_j} (1 - \epsilon_k) \prod_{(a,b) \equiv k \notin A_j} \epsilon_k \quad \forall j \equiv (a, B) \in \mathcal{H} \right\}.$$

h) State the hyperarc-hyperarc incidence matrix $\boldsymbol{Q}$.

i) Determine the broadcast capacity vector $\boldsymbol{y}$.

j) Explicitly state the lossy hyperarc flow bound $\boldsymbol{Nx} \leq \boldsymbol{y}$.

k) Enumerate all $s - t$ cuts S and their respective capacities $v(S_a)$ for $s = 1$ and $t = 3$.

l) State the min-cut capacity r for a flow from s to t in dependency of τ_1 and τ_2.

m) Determine τ_1 and τ_2 such that r is maximized.

We now consider the multicast $s = 1$ and $T = \{2, 3\}$.

n) Determine the missing $s - T$ cut and its capacity.

o) State the optimization problem to maximize the multicast capacity r'.

p) Determine the maximum multicast rate r'^* by solving the problem. Assume $\epsilon_4 = \epsilon_5$, otherwise the various cases are more complex.

Hint: It is sufficient to differentiate between cases and to express τ_2, τ_3 by means of τ_1. Except for the trivial case, the expression for τ_1 is not nice.

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$(a, B) \in \mathcal{H}$	$j \equiv (a, B)$	$\mathcal{A}_j$	A_j	z_j	y_j

Table 1: Fill in values from different subproblems.