draw Backoff slots $\rightarrow$ $x \in [0, C]$, $x \in \mathbb{N}$

1) a) $Pr[X = n] = \frac{1}{C+1}$

b) $Pr[X \leq n] = \sum_{i=0}^{n} Pr[X = i] = \frac{n+1}{C+1}$

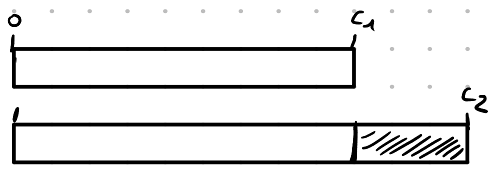
c) Nodes 1, 2 where $C_1 = C_2 = C$

$\left.\begin{array}{l} y \text{ slots} \\ x \text{ slots} \end{array}\right\}$ Collision iff $y = x$

$$Pr[y = x] = \sum_{i=0}^{C} Pr[X_1 = i \wedge X_2 = i]$$
$$= \sum_{i=0}^{C} Pr[X_1 = i] \, Pr[X_2 = i]$$
$$= \sum_{i=0}^{C} \frac{1}{C+1} \cdot \frac{1}{C+1} = \sum_{i=0}^{C} \left(\frac{1}{C+1}\right)^2$$
$$= (C+1) \cdot \left(\frac{1}{C+1}\right)^2 = \frac{1}{C+1}$$

d) Nodes 1, 2 choose x, y slots $\begin{array}{l} x \in [0, C_1] \\ y \in [0, C_2] \end{array} \Big| \; C_1 \neq C_2$

$$Pr[\text{"collision"}] = \sum_{i=0}^{\min(C_1, C_2)} Pr[X_1 = i \wedge X_2 = i]$$



let without loss of generality $C_2 \geq C_1$

$$\Rightarrow \sum_{i=0}^{C_1} Pr[X_1 = i] \cdot Pr[X_2 = i]$$
$$= \sum_{i=0}^{C_1} \frac{1}{C_1+1} \cdot \frac{1}{C_2+1} = (C_1+1) \cdot \frac{1}{(C_1+1)(C_2+1)}$$
$$= \frac{1}{C_2+1}$$

e) Node 1, 2

1 → $x \in [0, c_1]$

2 → $y \in [0, c_2]$

$c_1 \leq c_2$

should be $c_1 < c_2$, otherwise case $Pr[x=c_1]\,Pr[y>c_1]$ broken

Node 1 wins iff $x < y$

$$Pr[x<y] = \sum_{i=0}^{c_1} Pr[x=i] \underbrace{Pr[y>i]}_{\to\ 1 - Pr[y \leq i]}$$

$$= \sum_{i=0}^{c_1} \frac{1}{c_1+1}\left(1 - \frac{i+1}{c_2+1}\right)$$

$$= \sum_{i=0}^{c_1} \frac{1}{c_1+1} - \frac{i+1}{(c_1+1)(c_2+1)}$$

$$= \sum_{i=0}^{c_1} \frac{1 \cdot (c_2 \cancel{+1})}{(c_1+1)(c_2+1)} - \frac{i \cancel{+1}}{(c_1+1)(c_2+1)}$$

$$= \sum_{i=0}^{c_1} \frac{c_2}{(c_1+1)(c_2+1)} - \frac{i}{(c_1+1)(c_2+1)}$$

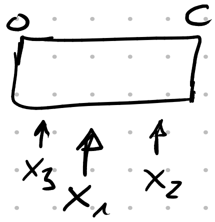
$$= \frac{\cancel{(c_1+1)} \cdot c_2}{\cancel{(c_1+1)}(c_2+1)} - \sum_{i=0}^{c_1} \frac{i}{(c_1+1)(c_2+1)} \quad \Bigg| \quad \sum_{i=0}^{c_1} x = \sum_{i=1}^{c_1}$$

$$= \frac{c_2}{c_2+1} - \frac{c_1 \cancel{(c_1+1)}}{2 \cancel{(c_1+1)}(c_2+1)}$$

$$= \frac{2(c_2)}{2(c_2+1)} - \frac{c_1}{2(c_2+1)} = \frac{2c_2 - c_1}{2(c_2+1)}$$

f) expected duration of cont. phase T_n for n Nodes where $C_i = C_j, \forall i,j \in X$ $\quad X = \{0, 1, \dots, c\}$

$$E[X] = \sum_{k=0}^{\infty} k \Pr[X=k] = \sum_{k=0}^{\infty} 1 - \Pr[X \le k] = \boxed{\sum_{k=0}^{\infty} \Pr[X > k]}$$



$$E[T_n] = E[\min\{X_i \mid i \in \{1, \dots, n\}\}]$$

$$= \sum_{k=0}^{c} \Pr[X_i > k \mid i \in \{1, \dots, n\}]$$

$$= \sum_{k=0}^{c} \prod_{j=1}^{n} \Pr[X_i > k] = \sum_{k=0}^{c} \prod_{j=1}^{n} 1 - \Pr[X \le k]$$

$$= \sum_{k=0}^{c} \prod_{j=1}^{n} 1 - \frac{k+1}{c+1} = \sum_{k=0}^{c} \prod_{j=1}^{n} \frac{1(c+1)}{c+1} - \frac{k+1}{c+1}$$

swap sum direction

$$= \sum_{k=0}^{c} \prod_{j=1}^{n} \frac{c-k}{c+1} = \sum_{k=0}^{c} \left(\frac{c-k}{c+1}\right)^n = \sum_{k=0}^{c} \left(\frac{k}{c+1}\right)^n$$

g) exp. dur. of cont. phase T for 2 nodes with $C_1 \neq C_2$

$$E[T] = E[\min\{X_1, X_2\}]$$

$$= \sum_{k=0}^{\min\{C_1, C_2\}} \Pr[X_1 > k \land X_2 > k]$$

$$= \sum_{k=0}^{\min\{C_1, C_2\}} \Pr[X_1 > k] \Pr[X_2 > k]$$

$$= \sum_{k=0}^{\min\{C_1, C_2\}} (1 - \Pr[X_1 \le k])(1 - \Pr[X_2 \le k])$$

$$= \sum_{k=0}^{\min\{C_1, C_2\}} \left(1 - \frac{k+1}{C_1+1}\right)\left(1 - \frac{k+1}{C_2+1}\right)$$

$$= \sum_{k=0}^{\min\{C_1, C_2\}} \left(\frac{C_1+1}{C_1+1} - \frac{k+1}{C_1+1}\right)\left(\frac{C_2+1}{C_2+1} - \frac{k+1}{C_2+1}\right)$$

$$= \sum_{k=0}^{\min\{C_1, C_2\}} \frac{C_1-k}{C_1+1} \cdot \frac{C_2-k}{C_2+1} = \dots$$