Chair of Network Architectures and Services
Department of Informatics
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Eexam
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Note:

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Network Coding

Exam:	IN2315 / Endterm	**Date:**	Friday 19^{th} February, 2021
Examiner:	Prof. Dr.-Ing. Georg Carle	**Time:**	14:15 – 15:45

Working instructions

- This exam consists of **12 pages** with a total of **4 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 60 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from __________ to __________ / Early submission at __________

Problem 1 **Finite fields (12 credits)**

Given the finite field $\mathbb{F}_p$, we consider the finite extension field

$$F_q[x] = \left\{ \sum_{i=0}^{n-1} a_i x^i \;\middle|\; a_i \in \mathbb{F}_p \right\}. \tag{1.1}$$

0
1

a)* State the conditions on p, q, and n such that a finite field $F_q[x]$ exists.

0
1

b)* Reason why there is an extension field for $p = 4$ and $n = 4$.

We now consider the binary extension field F_{256} with the reduction polynomial $r(x) = x^8 + x^4 + x^3 + x + 1$, and the two elements $a(x) = x^7 + x + 1$ and $b(x) = x^5 + 1$.

0
1

c)* Determine the product $a(x) \cdot b(x)$ in the given field using polynomial division.

0
1

d)* Discuss the disadvantages of the polynomial division with respect to performance when naively implemented.

Figure 1.1 shows the log and alog tables for the given field, which is also known from the lecture.

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	00	ff	19	01	32	02	1a	c6	4b	c7	1b	68	33	ee	df	03
1	64	04	e0	0e	34	8d	81	ef	4c	71	08	c8	f8	69	1c	c1
2	7d	c2	1d	b5	f9	b9	27	6a	4d	e4	a6	72	9a	c9	09	78
3	65	2f	8a	05	21	0f	e1	24	12	f0	82	45	35	93	da	8e
4	96	8f	db	bd	36	d0	ce	94	13	5c	d2	f1	40	46	83	38
5	66	dd	fd	30	bf	06	8b	62	b3	25	e2	98	22	88	91	10
6	7e	6e	48	c3	a3	b6	1e	42	3a	6b	28	54	fa	85	3d	ba
7	2b	79	0a	15	9b	9f	5e	ca	4e	d4	ac	e5	f3	73	a7	57
8	af	58	a8	50	f4	ea	d6	74	4f	ae	e9	d5	e7	e6	ad	e8
9	2c	d7	75	7a	eb	16	0b	f5	59	cb	5f	b0	9c	a9	51	a0
10	7f	0c	f6	6f	17	c4	49	ec	d8	43	1f	2d	a4	76	7b	b7
11	cc	bb	3e	5a	fb	60	b1	86	3b	52	a1	6c	aa	55	29	9d
12	97	b2	87	90	61	be	dc	fc	bc	95	cf	cd	37	3f	5b	d1
13	53	39	84	3c	41	a2	6d	47	14	2a	9e	5d	56	f2	d3	ab
14	44	11	92	d9	23	20	2e	89	b4	7c	b8	26	77	99	e3	a5
15	67	4a	ed	de	c5	31	fe	18	0d	63	8c	80	c0	f7	70	07

(a) Log

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	01	03	05	0f	11	33	55	ff	1a	2e	72	96	a1	f8	13	35
1	5f	e1	38	48	d8	73	95	a4	f7	02	06	0a	1e	22	66	aa
2	e5	34	5c	e4	37	59	eb	26	6a	be	d9	70	90	ab	e6	31
3	53	f5	04	0c	14	3c	44	cc	4f	d1	68	b8	d3	6e	b2	cd
4	4c	d4	67	a9	e0	3b	4d	d7	62	a6	f1	08	18	28	78	88
5	83	9e	b9	d0	6b	bd	dc	7f	81	98	b3	ce	49	db	76	9a
6	b5	c4	57	f9	10	30	50	f0	0b	1d	27	69	bb	d6	61	a3
7	fe	19	2b	7d	87	92	ad	ec	2f	71	93	ae	e9	20	60	a0
8	fb	16	3a	4e	d2	6d	b7	c2	5d	e7	32	56	fa	15	3f	41
9	c3	5e	e2	3d	47	c9	40	c0	5b	ed	2c	74	9c	bf	da	75
10	9f	ba	d5	64	ac	ef	2a	7e	82	9d	bc	df	7a	8e	89	80
11	9b	b6	c1	58	e8	23	65	af	ea	25	6f	b1	c8	43	c5	54
12	fc	1f	21	63	a5	f4	07	09	1b	2d	77	99	b0	cb	46	ca
13	45	cf	4a	de	79	8b	86	91	a8	e3	3e	42	c6	51	f3	0e
14	12	36	5a	ee	29	7b	8d	8c	8f	8a	85	94	a7	f2	0d	17
15	39	4b	dd	7c	84	97	a2	fd	1c	24	6c	b4	c7	52	f6	01

(b) Alog

Figure 1.1: Log and alog table for GF(256)

e)* Explain the log table approach.

0 1 2

f) Determine the product of $a'(x) = x^2 + x$ and $b'(x) = x^6 + x^4 + x + 1$ using the log table approach.

0 1 2 3 4

g) State an advantage and a disadvantage of the log table approach with respect to efficiency.

0 1

Problem 2 Metrics (12 credits)

We consider the wireless network depicted in Figure 2.1 consisting of nodes $N = (s, 1, 2, t)$. Per-node packet erasure probabilities are given $\forall i, j \in N$ as $0 \leq \epsilon_{ij} \leq 1$ and $i \neq j$. Erasures are assumed to be indepentently and identically distributed.

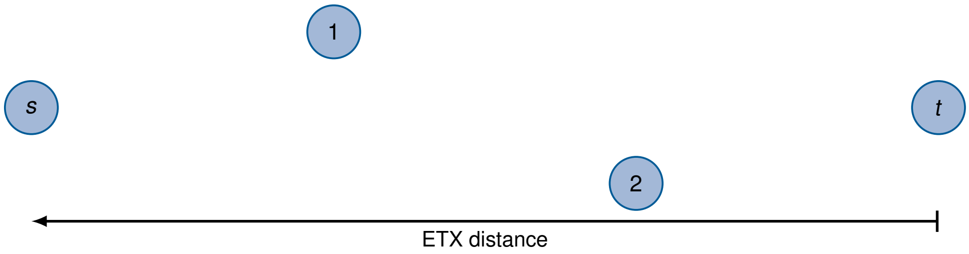

Figure 2.1: Wireless network, all hyperarcs are assumed to exist.

a)* Briefly explain the ETX distance between s and t.

b)* Argue which distribution the individual terms of the ETX metric adhere to.

In the following, we want to derive the amount of packets individual nodes have to transmit per source packet. To this end, we need the

$$R_j = \sum_{i>j} z_i(1 - \epsilon_{ij}), \tag{2.1}$$

$$L_j = \sum_{i>j} \left(z_i(1 - \epsilon_{ij}) \prod_{k<j} \epsilon_{ik} \right), \text{ and} \tag{2.2}$$

$$z_j = \frac{L_j}{1 - \prod_{k<j} \epsilon_{jk}}. \tag{2.3}$$

c)* Explain R_j as given in (2.1).

d)* Derive R_j for $j \in \{1, 2, t\}$. Note that $R_s = 1$.

0 1 2 3

e) Explain L_j as given in (2.2).

0 1

f)* Derive L_j for $j \in N$.

0 1 2

g)* Explain z_j as given in (2.3).

0 1

h) Derive z_j for $j \in N$.

0 1 2

Problem 3 Network coding in lossy wireless packet networks (22 credits)

We consider the network depicted by the hypergraph $G = (N, \mathcal{H})$ in Figure 3.1. Note that only maximum hyperarcs are drawn.

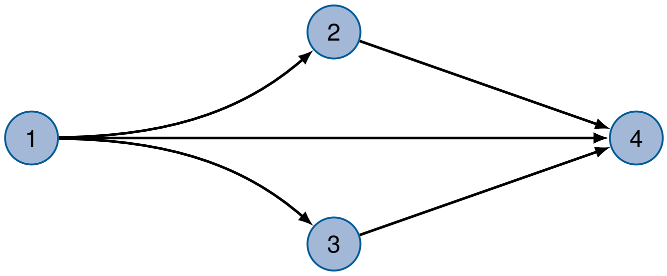

Figure 3.1: Hypergraph of example network, only maximum hyperarcs are drawn

We assume that packet losses, i. e., erasure events, are independently and identically distributed. Resource shares are denoted by $0 \leq \tau_i \leq 1$ for all $i \in N$. We further assume othrogonal medium access, i. e., nodes to not transmit concurrently.

a)* Draw the induced graph $G' = (N, A)$ and number the arcs in lexicographic order.

$(a, B) \in \mathcal{H}$	$j \equiv (a, B)$	z_j	y_j

Table 3.1: Solution table for Problems b) to d)

b)* List all hyperarcs $(a, B) \in \mathcal{H}$ in lexicographic order and assign arc indices $j \equiv (a, B)$ in Table 3.1.

c) Determine the network's hyperarc capacity region (Table 3.1).

d) Determine the network's broadcast capacity region (Table 3.1).

We now consider an unicast session between Node 1 and Node 4.

e) List all $s - t$ cuts.

f) Derive the value of each $s - t$ cut.

We now assume that $\epsilon_5 \geq \epsilon_3 \wedge \epsilon_4 < \epsilon_3 \wedge \epsilon_1 < 1$.

g)* Reason which nodes participate in forwarding traffic.

0
1
2

h) Given the conditions above, restate the cut values.

0
1
2

i) Reason which cuts are binding?

0
1
2

j) Determine τ_i for all $i \in N$ in that case.

Problem 4 Quiz (14 credits)

The following subproblems can be solved independently of each other.

a)* Assuming a file is available at three nodes. A fourth node requests the file. Each of the three nodes transmits a random linear combination (uniformely and identically distributed) using XOR only. Determine the decoding probability at the fourth node assuming that no packets are lost.

0
1
2

b)* The IEEE 802.11 header has (up to) four address fields. Briefly explaint the usage of those fields.

0
1
2

c)* Given a IEEE 802.11-based network. Explain the tradeoff between packet errors and frame size with respect to media access.

0
1

d)* A IEEE 802.11-based network under good conditions has about 2 % packet loss at the PHY. Explain (1) why TCP has problems with such kind of packet loss and (2) why TCP still works fine in that case.

0
1
2

e)* Given the incidence matrix ***M*** of network. Determine rank***M***.

0
1

0
1

f)* Given the incidence matrix $\boldsymbol{M}$ of network. Explain the intuition behind $\operatorname{rank} \operatorname{null} \boldsymbol{M}$.

0
1
2

g)* Given a fully connected wireless network with n nodes. Determine the total amount of possible hyperarcs.

0
1
2

h) Assuming that the link layer exposes a packet loss rate of 5 % to the network layer. Explain the effect on TCP.

0
1

i) Describe the hidden station problem.

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.