Chair of Network Architectures and Services
Department of Informatics
Technical University of Munich

Esolution
Place student sticker here

Note:
- During the attendance check a sticker containing a unique code will be put on this exam.
- This code contains a unique number that associates this exam with your registration number.
- This number is printed both next to the code and to the signature field in the attendance check list.

Network Coding

Exam:	IN2315 / Endterm	**Date:**	Tuesday 18th February, 2020
Examiner:	Prof. Dr.-Ing. Georg Carle	**Time:**	13:30 – 15:00

Sample Solution

Working instructions

- This exam consists of **12 pages** with a total of **5 problems**.
  Please make sure now that you received a complete copy of the exam.
- The total amount of achievable credits in this exam is 60 credits.
- Detaching pages from the exam is prohibited.
- Allowed resources:
  - one **cheatsheet (A4)**
  - one **non-programmable pocket calculator**
  - one **analog dictionary** English ↔ native language
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from ________ to ________	/	Early submission at ________	

Problem 1 Network flow problem (22 credits)

We consider the four-node wireless network represented as induced graph $G = (N, A)$ in Figure 1.1.

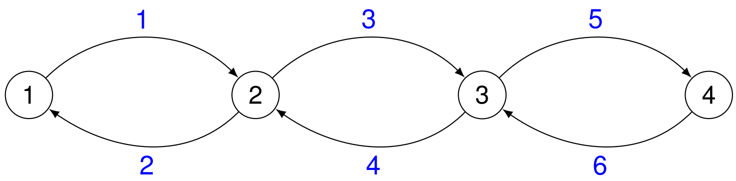

Figure 1.1: four-node network

We assume that packet losses, i. e., erasure events, are indepedently and identically distributed with expectation ϵ_k for all $k \in A$. Assume that all arcs $k \in A$ have unit capacity. Resource shares are denoted by $0 \leq \tau_i \leq 1$ for all $i \in N$. We further assume orthogonal medium access, i. e., nodes do not transmit concurrently. All nodes are in range of each other, i. e., a node senses ongoing transmissions of any other node although those transmissions may not be successfully overheard.

a)* Enumerate the arcs in Figure 1.1 in lexicographic order as known from the lecture.

b)* Determine the incidence matrix.

$$\boldsymbol{M} = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ -1 & 1 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}$$

c)* List all hyperarcs $(a, B) \in \mathcal{H}$ in lexicographic ascending order and assign numbers $j \equiv (a, B)$ in Table 1.1.

d)* List the set of induced arcs A_j for all $j \in H$ in Table 1.1.

e)* Determine the network's hyperarc capacity region (Table 1.1).

f) Determine the network's broadcast capacity region (Table 1.1).
We now consider a unicast session between nodes $s = 1$ and $t = 4$.

g)* Enumerate all $s - t$ cuts S and their respective capacities $v(S_a)$.

cut	capacity
$S_1 = \{1\}$	$v(S_1) = \tau_1(1 - \epsilon_1)$
$S_2 = \{1, 2\}$	$v(S_2) = \tau_2(1 - \epsilon_3)$
$S_3 = \{1, 3\}$	$v(S_3) = \tau_1(1 - \epsilon_1) + \tau_3(1 - \epsilon_5)$
$S_4 = \{1, 2, 3\}$	$v(S_4) = \tau_3(1 - \epsilon_5)$

Sample Solution

$(a, B) \in \mathcal{H}$	$j \equiv (a, B)$	A_j	z_j	y_j
$(1, \{2\})$	1	$\{(1, 2)\}$	$\tau_1(1 - \epsilon_1)$	$\tau_1(1 - \epsilon_1)$
$(2, \{1\})$	2	$\{(2, 1)\}$	$\tau_2(1 - \epsilon_2)\epsilon_3$	$\tau_2(1 - \epsilon_2)$
$(2, \{3\})$	3	$\{(2, 3)\}$	$\tau_2(1 - \epsilon_3)\epsilon_2$	$\tau_2(1 - \epsilon_3)$
$(2, \{1, 3\})$	4	$\{(2, 1), (2, 3)\}$	$\tau_2(1 - \epsilon_2)(1 - \epsilon_3)$	$\tau_2(1 - \epsilon_2\epsilon_3)$
$(3, \{2\})$	5	$\{(3, 2)\}$	$\tau_3(1 - \epsilon_4)\epsilon_5$	$\tau_3(1 - \epsilon_4)$
$(3, \{4\})$	6	$\{(3, 4)\}$	$\tau_3(1 - \epsilon_5)\epsilon_4$	$\tau_3(1 - \epsilon_5)$
$(3, \{2, 4\})$	7	$\{(3, 2), (3, 4)\}$	$\tau_3(1 - \epsilon_4)(1 - \epsilon_5)$	$\tau_3(1 - \epsilon_4\epsilon_5)$
$(4, \{3\})$	8	$\{(4, 3)\}$	$\tau_4(1 - \epsilon_6)$	$\tau_4(1 - \epsilon_6)$

Table 1.1: Fill in values from different subproblems. (An additional pre-print can be found on Page 12)

Sample Solution

h)* State the conditions on the loss probabilities ϵ_k such that communication from s to t is possible.

$\epsilon_1 < 1$
$\epsilon_3 < 1$
$\epsilon_5 < 1$

i)* Express the minimum number of packets to be transmitted by each node on average to transmit N packets.

$n_1 = \frac{N}{1-\epsilon_1}$
$n_2 = \frac{N}{1-\epsilon_3}$
$n_3 = \frac{N}{1-\epsilon_5}$

j) Express the resource shares considering all nodes in range.

$\tau_1 = \frac{n_1}{n_1+n_2+n_3}$
$\tau_2 = \frac{n_2}{n_1+n_2+n_3}$
$\tau_3 = \frac{n_3}{n_1+n_2+n_3}$

k) Express the resource shares considering nodes 1 and 3 are not in range of each other, i. e., assuming concurrent transmission of both nodes is possible.

$\tau_1 = \frac{n_1}{n_1+n_2}$
$\tau_2 = \min\{\frac{n_2}{n_1+n_2}, \frac{n_2}{n_2+n_3}\}$
$\tau_3 = \frac{n_3}{n_2+n_3}$

Sample Solution

Problem 2 Finite fields and finite extension fields (12 credits)

Given a finite field (Galois field) $\mathbb{F}_p \subset \mathbb{N}_0$ as introduced in the lecture, we seek to define a set of polynomials $F_q[x]$ such that $F_q[x]$ becomes a finite extension field over $\mathbb{F}_p$.

a)* State the condition on $p \in \mathbb{N}$ as well as the finite operators $< +, \cdot >$ such that $\mathbb{F}_p$ is a finite field.

p must be primeand the binary operations $+, \cdot$ must be taken modulo p.

b)* For which q is an extension field guaranteed to exist?

For all $q = p^n$ with $n \in \mathbb{N}$.

c) State the set of elements $F_q[x]$ of an extension field over $\mathbb{F}_p$.

$$F_q[x] = \left\{ \sum_{i=0}^{n-1} a_i x^i \,\middle|\, a_i \in \mathbb{F}_p \right\}.$$

d) Define the binary operators $< +, \cdot >$ such that $F_q[x]$ becomes an extension field.
Note: Take care to fully define all variables you use in your definition!

For any $a, b \in F_q[x]$ must hold ...

$$a + b := \sum_{i=0}^{n-1} a_i x^i + \sum_{i=0}^{n-1} b_i x^i = \sum_{i=0}^{n-1} (a_i + b_i \mod p)\, x^i$$

$$a \cdot b := \left(\sum_{i=0}^{n-1} a_i x^i \cdot \sum_{i=0}^{n-1} b_i x^i \right) \mod r(x) \text{ where } r(x) \notin F_q[x] \text{ is an irreducible polynomial of degree } n$$

Sample Solution

There are various different implementations for multiplication over binary extension fields. Common approaches are *full table lookups* and *log tables*.

0
1

e)* Briefly explain how *full table lookups* work.

Full table lookups: All multiplication results are stored verbose in a table.

0
1
2

f)* Briefly explain how *log table lookups* work.

Log table lookups: $\log(a \cdot b) = \log(a) + \log(b) \Rightarrow a \cdot b = \exp(\log(a \cdot b)) = \exp(\log(a) + \log(b))$, where log and exp denote the discrete logarithm and its inverse operation, respectively.

0
1
2

g) Argue for which binary extension fields the algorithm of Subproblems e) and f) are best suited.

For fields such as $F_4[x]$ or $F_{16}[x]$, the penalty of having two memory lookups per multiplication is too much which is why we won't use the log table approach here.

Multiplication tables for larger fields such as $F_{256}[x]$ and above may be too large to fit into caches, which is why the penalty of memory consumption outweighs the overhead of an additional memory lookup.

Sample Solution

Problem 3 Link quality estimations (12 credits)

A receiver receives eight packets with the following sequence numbers:

$s_1 = 0$ $\quad s_2 = 22$ $\quad s_3 = 68$ $\quad s_4 = 98$

$s_5 = 102$ $\quad s_6 = 126$ $\quad s_7 = 152$ $\quad s_8 = 201.$

a)* Calculate the number of accumulated received (p_k), accumulated lost (q_k), and lost-since-the-last-reception packets (z_k).

$p_1 = 1$ $\quad p_2 = 2$ $\quad p_3 = 3$ $\quad p_4 = 4$

$p_5 = 5$ $\quad p_6 = 6$ $\quad p_7 = 7$ $\quad p_8 = 8$

$q_1 = 0$ $\quad q_2 = 21$ $\quad q_3 = 66$ $\quad q_4 = 95$

$q_5 = 98$ $\quad q_6 = 121$ $\quad q_7 = 146$ $\quad q_8 = 194$

$z_1 = 0$ $\quad z_2 = 21$ $\quad z_3 = 45$ $\quad z_4 = 29$

$z_5 = 3$ $\quad z_6 = 23$ $\quad z_7 = 25$ $\quad z_8 = 48$

b) Calculate and update *EWMA* link quality estimation after every reception for given $\alpha = 0.97$ and initial estimation $EWMA[0] = 1.0$.

$EWMA[1] = EWMA[0]\alpha^{z_1} + (1 - \alpha) = 1.0$ $\quad EWMA[2] = EWMA[1]\alpha^{z_2} + (1 - \alpha) = 0.54$

$EWMA[3] = EWMA[2]\alpha^{z_3} + (1 - \alpha) = 0.16$ $\quad EWMA[4] = EWMA[3]\alpha^{z_4} + (1 - \alpha) = 0.10$

$EWMA[5] = EWMA[4]\alpha^{z_5} + (1 - \alpha) = 0.11$ $\quad EWMA[6] = EWMA[5]\alpha^{z_6} + (1 - \alpha) = 0.09$

$WWMA[7] = EWMA[6]\alpha^{z_7} + (1 - \alpha) = 0.07$ $\quad EWMA[8] = EWMA[7]\alpha^{z_8} + (1 - \alpha) = 0.05$

c)* Determine Δt to be able to estimate 1% of link quality, in packet rate of one packet per second.

100 s

Sample Solution

0 1 2 3 4

d) Calculate and update *WM-EWMA* link quality estimation after every reception for given $\alpha = 0.97$, initial estimation *WM-EWMA*[0] = 1.0, and for the previosly calculated Δt.

$$WMEWMA[1] = WMEWMA[0] = 1.0$$
$$WMEWMA[2] = WMEWMA[1] = 1.0$$
$$WMEWMA[3] = WMEWMA[2] = 1.0$$
$$WMEWMA[4] = WMEWMA[3] = 1.0$$
$$WMEWMA[5] = WMEWMA[4]\alpha + (1 - WMEWMA[4])(4/100) = 0.97$$
$$WMEWMA[6] = WMEWMA[5] = 0.97$$
$$WMEWMA[7] = WMEWMA[6] = 0.97$$
$$WMEWMA[8] = WMEWMA[7]\alpha + (1 - WMEWMA[4])(3/100) = 0.94$$

Hint: First 4 packets are received in the first slot, next 3 packets are received in the second slot and the last packet is received in the last slot considering a slot is 100 seconds, packet rate is 1 packet per second and the sequence numbers of the received packets.

Sample Solution

Problem 4 ETX and EOTX metric (9 credits)

We consider the network depicted in Figure 4.1 that consists of four nodes $N = \{1, 2, 3, 4\}$ ordered according to their EOTX distance $d_{ij}\ \forall i, j \in N$ from left to right in ascending order. The erasure probabilities between nodes are considered independentely and identically desitrbuted with expectation $0 < \epsilon_{ij} < 1\ \forall i, j \in N$.

Figure 4.1: Four-node network. We consider all possible hyperarcs being present.

The EOTX distance d_{ij} between nodes $i, j \in N$ is known from the lecture to be

$$d_{ij} = \underbrace{\frac{1}{1 - \prod_{k<i} \epsilon_{ik}}}_{a)} + \underbrace{\sum_{k<i} d_{kj}(1 - \epsilon_{ik}) \prod_{l<k} \epsilon_{il}}_{b)} . \tag{1}$$

a)* Explain in your own words the meaning the of the first summand in (1).

Number of transmissions made by i to ensure that at least one node closer to j than i itself receives it.

b)* Explain in your own words the meaning the of the second summand in (1).

Number of transmissions made by a downstream node k privded that

- k overheard the transmission by k and
- no node closer to j than k also received that transmission.

c)* Derive d_{12}.

$$d_{12} = \frac{1}{1 - \epsilon_{12}}$$

Sample Solution

0
1

d) Derive d_{13}.

$$d_{13} = \frac{1}{1-\epsilon_{12}\epsilon_{13}} + d_{23}(1-\epsilon_{12})\epsilon_{13}$$
$$= \frac{1}{1-\epsilon_{12}\epsilon_{13}} + \frac{1}{1-\epsilon_{23}}(1-\epsilon_{12})\epsilon_{13}$$

0
1
2

e) Derive d_{14}.

$$d_{14} = \frac{1}{1-\epsilon_{12}\epsilon_{13}\epsilon_{14}} + d_{24}(1-\epsilon_{12})\epsilon_{13}\epsilon_{14} + d_{34}(1-\epsilon_{13})\epsilon_{14}$$
$$d_{24} = \frac{1}{1-\epsilon_{23}\epsilon_{24}} + d_{34}(1-\epsilon_{23})\epsilon_{24}$$
$$d_{34} = \frac{1}{1-\epsilon_{34}}$$

Sample Solution

Problem 5 Quiz (5 credits)

Each of the following subproblems can be solved independently of each other.

a)* Explain the difference between network coding and forward error correction.

0 1

With FEC coding / decoding is done at source / destination (or hop-to-hop). With network coding, intermediate nodes recode without necessarily decoding.

b)* Give an example of bidirectional network coding.

0 1

A wireless network with with at least one relay node between source and destination that combined packets from both flows (directions).

c)* In multicast networks, state the reasons for maximum flow increase through store-forward, tree-based forwarding and network coding approaches both formally (in terms of constraints) and practically (in terms of actions in the nodes).

0 1 2

store-forward vs tree-based forwarding

formal: by relaxing flow conservation constraint.

practical: by allowing intermediate nodes duplicate packets.

tree-based forwarding vs network coding

formal: by relaxing joint capacity constraint.

practical: by allowing intermediate nodes encode packets.

d)* How are control frames prioritized over data frames when using the distributed coordination function (DCF) in IEEE 802.11?

0 1

Control frames are sent after SIFS, which is shorter than DIFS.

Sample Solution

$(a, B) \in \mathcal{H}$	$j \equiv (a, B)$	A_j	z_j	y_j
$(1, \{2\})$	1	$\{(1, 2)\}$	$\tau_1(1 - \epsilon_1)$	$\tau_1(1 - \epsilon_1)$
$(2, \{1\})$	2	$\{(2, 1)\}$	$\tau_2(1 - \epsilon_2)\epsilon_3$	$\tau_2(1 - \epsilon_2)$
$(2, \{3\})$	3	$\{(2, 3)\}$	$\tau_2(1 - \epsilon_3)\epsilon_2$	$\tau_2(1 - \epsilon_3)$
$(2, \{1, 3\})$	4	$\{(2, 1), (2, 3)\}$	$\tau_2(1 - \epsilon_2)(1 - \epsilon_3)$	$\tau_2(1 - \epsilon_2\epsilon_3)$
$(3, \{2\})$	5	$\{(3, 2)\}$	$\tau_3(1 - \epsilon_4)\epsilon_5$	$\tau_3(1 - \epsilon_4)$
$(3, \{4\})$	6	$\{(3, 4)\}$	$\tau_3(1 - \epsilon_5)\epsilon_4$	$\tau_3(1 - \epsilon_5)$
$(3, \{2, 4\})$	7	$\{(3, 2), (3, 4)\}$	$\tau_3(1 - \epsilon_4)(1 - \epsilon_5)$	$\tau_3(1 - \epsilon_4\epsilon_5)$
$(4, \{3\})$	8	$\{(4, 3)\}$	$\tau_4(1 - \epsilon_6)$	$\tau_4(1 - \epsilon_6)$

Table 5.1: Additional pre-print for Problem 1

Sample Solution