Chair of Network Architectures and Services
Department of Informatics
Technical University of Munich

Note:
- During the attendance check a sticker containing a unique code will be put on this exam.
- This code contains a unique number that associates this exam with your registration number.
- This number is printed both next to the code and to the signature field in the attendance check list.

Network Coding

Exam:	IN2315 / Endterm	**Date:**	Wednesday 14th February, 2018
Examiner:	Prof. Dr.-Ing. Georg Carle	**Time:**	10:30 – 11:45

	P 1	P 2	P 3	P 4
I				
II				

Working instructions

- This exam consists of
  - **16 pages** with a total of **4 problems** and
  - a two-sided printed **cheat sheet**.

  Please make sure now that you received a complete copy of the exam.
- Detaching pages from the exam is prohibited.
- Subproblems marked by * can be solved without results of previous subproblems.
- **Answers are only accepted if the solution approach is documented.** Give a reason for each answer unless explicitly stated otherwise in the respective subproblem.
- Do not write with red or green colors nor use pencils.
- The total amount of achievable credits in this exam is 60 credits.
- Allowed resources:
  - one **analog dictionary** English ↔ native language
  - one **self-written cheat sheet** (A4 double-sided)
- Physically turn off all electronic devices, put them into your bag and close the bag.

Left room from ________ to ________ / Early submission at ________

Problem 1 IEEE 802.11 wireless networks (11 credits)

In this problem we consider an ordinary IEEE 802.11-based network as depicted in Figure 1.1. The two wireless devices and the access point are operating in infrastructure mode. The access points connects the wireless network to a Ethernet-based local network. The whole network makes up a single subnet.

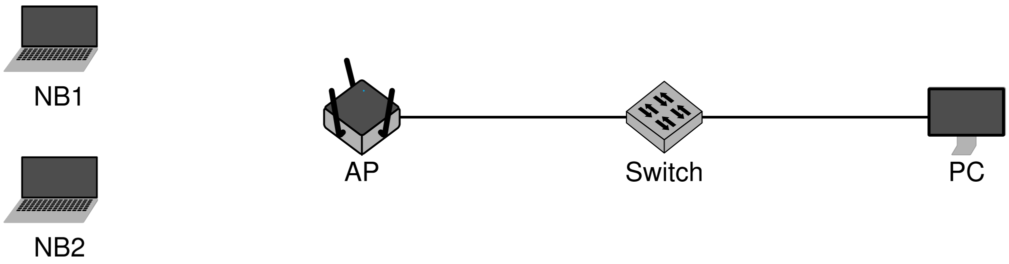

Figure 1.1: IEEE 802.11-based wireless network

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a)* What is the difference between *collision detection* and *collision avoidance* with respect to medium access?

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b)* Explain two major differences between Ethernet header and the (generic) IEEE 802.11 header.

c)* Name the three major frame types used by IEEE 802.11 and give one example for each type.

0 1 2 3

d)* Explain how a frame from NB1 to NB2 is being transmitted.

0 1 2

e)* Assuming that NB1 wants to communicate with PC. State the MAC addresses of the frame

1. between NB1 and the AP, and
2. between the AP and PC.

0 1 2

Hint: You may simply write a node's name as its MAC address, e. g. `NB1` to denote the MAC address of node NB1.

Problem 2 Finite extension fields (14 credits)

Given the finite field $\mathbb{F}_p$, we consider the finite extension field

$$F_q[x] = \left\{ \sum_{i=0}^{n-1} a_i x^i \mid a_i \in \mathbb{F}_p \right\}. \tag{1}$$

0 1 a)* State the conditions on p, q, and n such that a finite extension field $F_q[x]$ exists.

We now consider the specific extension field $F_q[x]$ resulting from $p = 5$ and $n = 2$.

0 1 2 b)* Find a generator of $\mathbb{F}_5$ and give a proof for your choice.

0 1 2 c)* List all elements of $F_q[x]$.

0 1 2 d)* Explain the purpose of a *reduction polynomial*.

0 1 2 e)* State the conditions a reduction polynomial has to fullfill.

f) Find a reduction polynomial for $F_q[x]$ and give a proof for your choice.

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Problem 3 Reformulating optimization problems (8 credits)

We consider the max-flow problem on a hypergraph $G = (V, H)$ in the lossy hyperarc model as stated in (2). As introduced in class and tutorials, $\boldsymbol{x}$ denotes the flow vector, $\boldsymbol{M}$ the incidence matrix describing the induced graph, $\boldsymbol{d}$ the demand vector, $\boldsymbol{N}$ the hyperarc-arc incidence matrix, $\boldsymbol{y}$ the broadcast capacity vector, and $\mathcal{Y}$ the broadcast capacity region.

$$\begin{aligned} \max r \quad \text{s.t.} \quad \boldsymbol{x} &\geq \boldsymbol{0} \\ \boldsymbol{M}\boldsymbol{x} &= r\boldsymbol{d} \\ \boldsymbol{N}\boldsymbol{x} &\leq \boldsymbol{y} \\ \boldsymbol{y} &\in \mathcal{Y} \end{aligned} \tag{2}$$

We want to reformulate this optimization problem to a form suitable for Matlab:

$$\begin{aligned} \min \boldsymbol{f}^T\boldsymbol{\xi} \quad \text{s.t.} \quad \boldsymbol{A}\boldsymbol{\xi} &\leq \boldsymbol{b} \\ \boldsymbol{A}^{'}\boldsymbol{\xi} &= \boldsymbol{b}^{'} \\ \boldsymbol{c} \leq \boldsymbol{\xi} &\leq \boldsymbol{c}^{'} \end{aligned} \tag{3}$$

The $\in$ (element of) relation cannot directly be represented by the canonical form given in (3). We therefore introduce the vector $\boldsymbol{\tau}$ of time shares as well as the constraints $\boldsymbol{1}^T\boldsymbol{\tau} \leq 1$ and $\boldsymbol{\tau} \geq \boldsymbol{0}$, which are implied by the broadcast capacity region $\mathcal{Y}$.

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a)* Find a matrix $\boldsymbol{Y} \in \mathbb{R}^{|H|\times|V|}$ such that $\boldsymbol{y} \leq \boldsymbol{Y}\boldsymbol{\tau}$ becomes an equivalent constraint to $\boldsymbol{y} \in \mathcal{Y}$.
Hint: Elements Y_{jv} of $\boldsymbol{Y}$ represent probabilities.

For the reformulation we collect all optimization variables into a single vector, which is defined as $\boldsymbol{\xi} = \begin{bmatrix} \boldsymbol{x} \\ \boldsymbol{\tau} \\ r \end{bmatrix}$.

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b)* Rewrite the equality constraint $\boldsymbol{M}\boldsymbol{x} = r\boldsymbol{d}$ to the form $[...]\boldsymbol{\xi} = \boldsymbol{0}$.

c)* Rewrite the inequality constraint $\boldsymbol{N}\boldsymbol{x} \leq \boldsymbol{Y}\boldsymbol{\tau}$ to the form $[...]\boldsymbol{\xi} \leq \mathbf{0}$.

d)* Rewrite the inequality constraint $\mathbf{1}^T\boldsymbol{\tau} \leq 1$ to the form $[...]\boldsymbol{\xi} \leq 1$.

e) Combine the results from Subproblems b) – d) to the canonical flow problem as given in (3).

$$\min \begin{bmatrix} \quad \end{bmatrix}^T \boldsymbol{\xi} \qquad \text{s. t.} \quad \begin{bmatrix} \qquad \end{bmatrix} \boldsymbol{\xi} \leq \begin{bmatrix} \quad \end{bmatrix}$$

$$\begin{bmatrix} \qquad \end{bmatrix} \boldsymbol{\xi} = \mathbf{0}$$

$$\boldsymbol{\xi} \geq \begin{bmatrix} \quad \end{bmatrix}$$

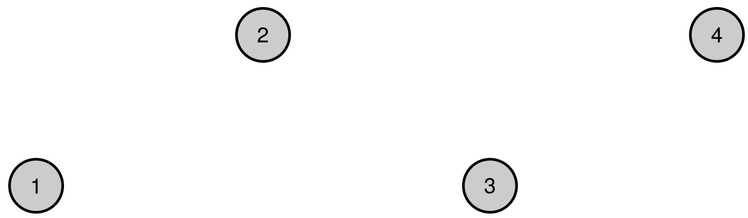
Problem 4 Network coding in lossy wireless packet networks (27 credits)

We consider the network represented as induced graph $G = (N, A)$ as defined by its incidence matrix

$$\boldsymbol{M} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 1 & 0 \\ 0 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 & -1 \end{bmatrix}. \tag{4}$$

We assume that packet losses, i. e., erasure events, are indepedently and identically distributed with expectation ε_k for all $k \in A$. Assume that all arcs $k \in A$ have unit capacity. Resource shares are denoted by $0 \leq \tau_i \leq 1$ for all $i \in N$. We further assume orthogonal medium access, i. e., nodes do not transmit concurrently.

a) Draw the induced graph $G = (N, \mathcal{A})$ and assign indices $k \in A$ to all arcs in lexicographic order.
Hint: An additional preprint can be found in Figure 4.1.



b)* List all hyperarcs $(a, B) \in \mathcal{H}$ in lexicographic ascending order and assign numbers $j \equiv (a, B)$ in Table 4.1.

c)* List the set of induced arcs A_j for all $j \in H$ in Table 4.1.

d)* Determine the network's hyperarc capacity region (Table 4.1).

e) Determine the network's broadcast capacity region (Table 4.1).

We now consider a unicast session between nodes 1 and 4.

f)* List all $s - t$ cuts.

g) Determine the value of each $s - t$ cut.

$(a, B) \in \mathcal{H}$	$j \equiv (a, B)$	A_j	z_j	y_j

Table 4.1: Fill in values from different subproblems. (An additional preprint can be found in Table 4.2)

For some specific ε_k, the cut capacities become

$$v(S_1) = \frac{3}{4}\tau_1,\ v(S_2) = \frac{1}{2}\tau_1 + \frac{3}{4}\tau_2,\ v(S_3) = \frac{1}{2}\tau_1 + \frac{7}{8}\tau_3,\ \text{and}\ v(S_4) = \frac{1}{2}\tau_2 + \frac{7}{8}\tau_3.$$

h)* What is the effect on $v(S_3)$ and $v(S_4)$ if τ_1 and τ_2 are being increased?

i)* Which condition must hold for τ_i with $i \in N$?

j)* Assuming that $v(S_1) = v(S_2)$ must hold for an optimal solution, determine τ_1, τ_2, and τ_3.

The correct solution of the previous subroblem is $\tau_1 = 1/2$, $\tau_2 = 1/6$, and $\tau_3 = 1/3$. (Don't even think about using these results to find the correct solution.)

k)* Show that this solution is indeed optimal.

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$(a, B) \in \mathcal{H}$	$j \equiv (a, B)$	A_j	z_j	y_j

Table 4.2: Additional preprint for Table 4.1

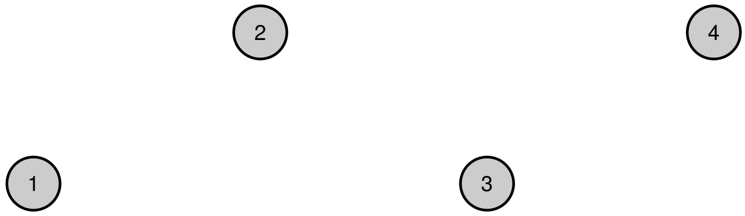

Figure 4.1: Additional preprint for Problem 4 a)

Additional space for solutions–clearly mark the (sub)problem your answers are related to and strike out invalid solutions.